CMB and non-Gaussianity

Enrique Martínez-González
Instituto de Física de Cantabria
Santander (Spain)
Outline

• Beyond the power spectrum
• Inflation model
• Isotropy and Gaussianity:
  – Non-standard inflationary models
  – Topological defects
  – Geometry and topology
  – Primordial magnetic fields
• Secondary anisotropies and contaminants:
  – Combined ISW-lensing effect
  – Extragalactic sources
• Summary
Planck will provide a nearly cosmic-variance-limited $C_l$ in the multipole range $2 < l < 2000$ where primary anisotropies dominate.
The CMB represents a unique observation to understand the universe.
The power spectrum and beyond

- Two fundamental properties of the cosmological paradigm are the homogeneity and isotropy of the matter-energy content of the universe and the Gaussianity of the initial perturbations.

- Those fundamental properties can be tested with precise observations of the CMB anisotropies. Due to the linear processes involved in the generation of those anisotropies the CMB test is straightforward. Cosmological studies based on e.g. the galaxy distribution or gravitational lensing are complicated by non-linear phenomena, as gravitational collapse or the bias, ...

- Under the assumption of Gaussianity and isotropy all the information is contained in the power spectrum of the CMB anisotropies.

- Testing Gaussianity and isotropy is important because:
  - It justifies the assumptions made in the process of deriving the cosmological parameters from the power spectrum.
  - It probes new physics and new scenarios of the early universe and of the geometry of the universe.
  - It allows a better understanding of secondary phenomena giving rise to deviations from Gaussianity.
Inflationary paradigm

Condition for accelerated expansion: \( \rho + 3p < 0 \)

Single scalar field:
\[
\rho_\phi = \frac{1}{2} \left( \frac{d\phi}{dt} \right)^2 + V(\phi) \quad p_\phi = \frac{1}{2} \left( \frac{d\phi}{dt} \right)^2 - V(\phi)
\]

\[\left( \frac{d\phi}{dt} \right)^2 < V(\phi) \quad \Rightarrow \quad \text{Accelerated expansion:} \quad a(t) \approx a_0 \exp[H(t - t_0)]\]
CONNECTION BETWEEN INFLATIONARY PARAMETERS AND OBSERVATIONS

- The slow-roll parameters of inflation are connected to observations as:
  - Fluctuation spectrum: \( \Delta = A k^n \)
  - \( n_s = 1 - 6 \varepsilon + 2 \eta \approx 1 \)
  - \( n_t = -2 \varepsilon \)
  - \( r \equiv \frac{A_t^2}{A_s^2} = 16 \varepsilon \)
  - \( \varepsilon = \frac{M_p^2}{2} \left( \frac{V'}{V} \right)^2 \)
  - \( \eta = M_p^2 \left( \frac{V''}{V} \right) \)
PREDICTIONS FROM INFLATION

- Generation of the initial energy-density fluctuations of the adiabatic type
- Nearly scale-invariant spectrum of fluctuations: scalar spectral index $n_s \leq 1$ (0.92-0.98)
- Generation of the Gravitational Wave Background $\Rightarrow$ existence of B-mode polarization

From WMAP and LSS: $n_s \approx 0.96 \Rightarrow 3\varepsilon - \mu \approx 0.02$

$r < 0.2 \Rightarrow \varepsilon < 0.01, V^{1/4} < 2.8 \times 10^{16} \text{GeV}$
Probing isotropy and Gaussianity

• Planck and the next generation of CMB polarization missions (e.g. COrE) will assess two important features of the current cosmological paradigm: the isotropy of the universe and the isotropy and Gaussianity of the primordial perturbations.

• There are many physical effects that might give rise to different deviations from isotropy and/or Gaussianity. The deviations might be classified according to their physical nature and origin as follows:
  – Non-standard inflationary models
  – Topological defects: cosmic strings, textures, …
  – Geometry and topology: Bianchi models, non-trivial topologies
  – Primordial magnetic fields
NON-STANDARD INFLATIONARY MODELS

- Multi-field scenarios (curvatron)
- Inhomogeneous reheating scenarios
- In these cases the GWB amplitude is expected to be undetectable and also $n_s \approx 1$ with high precision: it is not possible to extract information from the slow-roll parameters.
- There is a third observable: non-Gaussianity.

It is expected in the curvature perturbations at the second order:

$$\phi = \phi_L + f_{NL} \otimes \phi_L^2 + g_{NL} \otimes \phi_L^3$$

- $f_{NL}$ is the non-linear coupling parameter at first order and
- $g_{NL}$ is the one at second order.
Non-standard inflationary models (cont.)

• Large primordial non-Gaussianity can be generated if any of the conditions of canonical inflation is violated.

• The non-Gaussianity appears in higher-order moments like the bispectrum of the primordial perturbations of the gravitational potential \( \phi(k) \):

\[
\langle \phi(k_1)\phi(k_2)\phi(k_3) \rangle = (2\pi)^3 \delta^3(k_1 + k_2 + k_3) f_{NL} F(k_1,k_2,k_3)
\]

• Types of non-Gaussianity:
  – Local: \( k_1 \ll k_2 \approx k_3 \) e.g. multfield inflation, curvaton model, inhomogeneous reheating, hybrid inflation.
  – Equilateral: \( k_1 \approx k_2 \approx k_3 \) e.g. DBI inflation, ghost inflation.
  – Orthogonal: orthogonal to the local and equilateral cases.

• From WMAP 7-years data: (Komatsu et al. 2011)

\[
\begin{align*}
f_{NL}^{\text{local}} &= 32 \pm 21 \\
f_{NL}^{\text{equil}} &= 26 \pm 140 \\
f_{NL}^{\text{orthog}} &= -202 \pm 104
\end{align*}
\]

• Consistent results obtained with other techniques (e.g. wavelets, neural networks): Curto et al. 2011, Casaponsa et al. 2011

• A \( f_{NL} \) detection of the local type would rule out all single-field inflation models! (Creminelli & Zaldarriaga 2004)
Shape functions: local, equilateral, orthogonal
The bispectrum

- The bispectrum represents the lowest-order correlator to probe non-Gaussianity. Most models of inflation make specific predictions for it. It is defined by:

\[ B_{m_1m_2m_3}^{\ell_1\ell_2\ell_3} = \left\langle a_{\ell_1m_1} a_{\ell_2m_2} a_{\ell_3m_3} \right\rangle \]

- Most theories assume isotropy and therefore the angular average bispectrum is usually considered:

\[ B_{\ell_1\ell_2\ell_3} = \sum_{m_1m_2m_3} \left( \begin{array}{c} \ell_1 \ell_2 \ell_3 \\ m_1m_2m_3 \end{array} \right) \left\langle a_{\ell_1m_1} a_{\ell_2m_2} a_{\ell_3m_3} \right\rangle \]

- Another convenient quantity is the reduced bispectrum:

\[ B_{m_1m_2m_3}^{\ell_1\ell_2\ell_3} = G_{m_1m_2m_3}^{\ell_1\ell_2\ell_3} b_{\ell_1\ell_2\ell_3} \]

where \( G_{m_1m_2m_3}^{\ell_1\ell_2\ell_3} \) is the Gaunt integral.
**$f_{NL}$ estimation**

- In the limit of weak non-Gaussianity the optimal bispectrum estimator for $f_{nl}$ can be obtained by performing an Edgeworth expansion of the pdf (Babich 2005):

$$P(a \mid f_{nl}) = \left[ 1 - f_{nl} \sum_{\ell,m_1} \langle a_{\ell,m_1} a_{\ell_2 m_2} a_{\ell_3 m_3} \rangle_{f_{nl}} \right] \frac{\partial}{\partial a_{\ell,m_1}} \frac{\partial}{\partial a_{\ell_2 m_2}} \frac{\partial}{\partial a_{\ell_3 m_3}} + O(f_{nl}^2) \exp \left( -\frac{1}{2} \sum_{\ell,m_i} a_{\ell m_i}^* C^{-1}_{\ell m_4, \ell_3 m_5} a_{\ell_5 m_5} \right) \left( \frac{2\pi}{\text{det}(C)} \right)^{N/2}$$

$$C = C^{CMB} + C^{\text{noise}}$$ is the signal plus noise power spectrum.

- A necessary and sufficient condition to saturate the Cramer-Rao inequality is given by the condition:

$$\frac{\partial \log P(a \mid f_{nl})}{\partial f_{nl}} = F_{f_{nl}f_{nl}}(a) \left( ^{\wedge}f_{nl} - f_{nl} \right)$$

where $F$ is the fisher matrix:

$$F_{f_{nl}f_{nl}}(a) = \left\langle \frac{\partial^2 \ln p(a \mid f_{nl})}{\partial f_{nl}^2} \right\rangle$$
The optimal $f_{\text{NL}}$ estimator has been shown to be (Babich 2005, Creminelli et al. 2006):

$$
\hat{f}_{\text{NL}} = \frac{1}{N} \sum_{\ell, m} \left\langle a_{\ell m_1} a_{\ell m_2} a_{\ell m_3} \right\rangle f_{\text{nl}} = 1 \left( C_{\ell m_1, \ell m_2}^{-1} C_{\ell m_3, \ell m_4}^{-1} a_{\ell m_1} a_{\ell m_2} a_{\ell m_3} - 3 C_{\ell m_1, \ell m_2}^{-1} C_{\ell m_3, \ell m_4}^{-1} a_{\ell m_1} a_{\ell m_2} a_{\ell m_3} \right)
$$

However, for some real situations involving millions of data and complex masks and noise the optimal estimator may be unfeasible to compute and other reduced estimators can be more convenient (e.g. KSW, wavelet, modal, binned).

The KSW (Komatsu, Spergel and Wandelt 2005) assumes a diagonal covariance matrix and when generalized with a linear term takes the form (Creminelli et al. 2006):

$$
\hat{f}_{\text{NL}} = \frac{1}{N} \sum_{\ell, m} \left\langle a_{\ell m_1} a_{\ell m_2} a_{\ell m_3} \right\rangle f_{\text{nl}} = 1 \left( C_{\ell_1 \ell_2 \ell_3}^{-1} a_{\ell_1 m_1} a_{\ell_2 m_2} a_{\ell_3 m_3} - 3 C_{\ell_1 \ell_2 \ell_3}^{-1} a_{\ell_1 m_1} a_{\ell_2 m_2} a_{\ell_3 m_3} \right)
$$
The wavelet estimator

- Wavelets are very useful analysing tools for their spatial-scale localization.
- The Spherical Mexican Hat Wavelets (SMHW), defined in the plane by the second derivative of the Gaussian and projected to the sphere by a stereographic projection, are very compact in both real and harmonic spaces.
- They also show a good decorrelation of their coefficients at distances above the scale of the wavelet.
- From the wavelet coefficients $w(\vec{b}, R_i)$ (mean subtracted) we can construct the third order statistics:

$$q_{ijk} = \frac{1}{\sigma_i \sigma_j \sigma_k} \int w(\vec{b}, R_i) w(\vec{b}, R_j) w(\vec{b}, R_k) \, d\vec{b}$$

and combine them in a $\chi^2$ test:

$$\chi^2(f_{nl}) = \sum_{ijk, rst} (q_{ijk} - \langle q_{ijk} \rangle_{f_{nl}}) C^{-1}_{ijk, rst} (q_{rst} - \langle q_{rst} \rangle_{f_{nl}})$$

The estimator is

$$\hat{f}_{NL} = \sum_{ijk, rst} \langle q_{ijk} \rangle_{f_{NL} = 1} C^{-1}_{ijk, rst} \langle q_{rst} \rangle_{f_{NL} = 1}$$
SMHW properties
SMHW properties
Linear term

- The variance of a cubic estimator can be improved if a linear term is included (Wick’s product).
- A linear term naturally appears in the derivation of the optimal estimator

\[
\hat{f}_{NL} = \frac{1}{N} \sum_{\ell,m} \left\{ B_{l_{1}l_{2},l_{3}}^{f_{NL}} \left( \begin{array}{ccc} l_{1} & l_{2} & l_{3} \\ m_{1} & m_{2} & m_{3} \end{array} \right) \right\} \left\{ (C^{-1}a)_{l_{1}m_{1}} (C^{-1}a)_{l_{2}m_{2}} (C^{-1}a)_{l_{3}m_{3}} - 3C^{-1}_{l_{1}m_{1},l_{2}m_{2}} (C^{-1}a)_{l_{3}m_{3}} \right\}
\]

and it is incorporated when a diagonal covariance is assumed (Creminelli et al. 2006)

\[
\hat{f}_{NL} = \frac{1}{N} \sum_{\ell,m} \left\{ \langle a_{\ell_{1}m_{1}} a_{\ell_{2}m_{2}} a_{\ell_{3}m_{3}} \rangle_{f_{NL}=1} \left( C^{-1}_{\ell_{1}} C^{-1}_{\ell_{2}} C^{-1}_{\ell_{3}} a_{\ell_{1}m_{1}} a_{\ell_{2}m_{2}} a_{\ell_{3}m_{3}} - 3C^{-1}_{\ell_{1},l_{2}m_{2}} C^{-1}_{\ell_{3}} C_{\ell_{1},l_{2}m_{2}} a_{\ell_{3}m_{3}} \right) \right\}
\]

- For all-sky and isotropic noise the linear term is the monopole and thus zero.
- For mads and noise anisotropies similar to those of WMAP or Planck the linear term is of the order of the cubic term.
- For the SMHW a linear term can be also incorporated and it is equivalent to subtract the mean value of the wavelet coefficients at each scale (Curto et al. 2011). However for needlets the contribution is not negligible (Donzelli et al. 2012).
WMAP KQ75 mask and V+W anisotropic noise
The wavelet estimator has been proved to be optimal (within a few percent) and has provided results consistent with the optimal harmonic estimator for WMAP (Curto et al. 2011).

The estimate of the contribution of point sources have been studied with this technique using the de Zotti et al. model for radio galaxies:

\[ \Delta f_{NL}^{\text{local}} = 2.5 \pm 3 \Rightarrow f_{NL}^{\text{local}} = 30 \pm 22 \]
\[ \Delta f_{NL}^{\text{equil}} = 37 \pm 18 \Rightarrow f_{NL}^{\text{equil}} = -90 \pm 146 \]
\[ \Delta f_{NL}^{\text{orthog}} = 25 \pm 14 \Rightarrow f_{NL}^{\text{orthog}} = -180 \pm 107 \]
Planck sensitivity to $f_{NL}$

(1-year of data)

(Curto, M-G, Barreiro 2011)
Including polarization

• The general bispectrum for temperature $T$ and polarization $E$ is defined as:

$$B_{\ell_1\ell_2\ell_3,m_1m_2m_3}^{pqr} = \left\langle a_{\ell_1m_1}^p a_{\ell_2m_2}^q a_{\ell_3m_3}^r \right\rangle$$

where $p,q,r=T,E$. Similarly for the angular-averaged bispectrum.

$$B_{\ell_1\ell_2\ell_3}^{pqr} = \sum_{m_1m_2m_3} \left( \begin{array}{ccc} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{array} \right) \left\langle a_{\ell_1m_1}^p a_{\ell_2m_2}^q a_{\ell_3m_3}^r \right\rangle$$

• The same procedure that was followed to derive the bispectrum estimator for only $T$ applies also to $(T,E)$. In particular optimal errors can be estimated by the Fisher matrix:

$$\frac{1}{\sigma^2} = \sum_{ijk,pqr} \sum_{\ell_1 \leq \ell_2 \leq \ell_3} \frac{1}{\Delta_{\ell_1\ell_2\ell_3}} B_{\ell_1\ell_2\ell_3}^{ijk} \left( Cov^{-1} \right)_{ijk,pqr} B_{\ell_1\ell_2\ell_3}^{pqr}$$

where

$$\Delta_{\ell_1\ell_2\ell_3} = 1 + 2\delta_{\ell_1\ell_2} \delta_{\ell_2\ell_3} + \delta_{\ell_1\ell_2} + \delta_{\ell_2\ell_3} + \delta_{\ell_1\ell_3}$$
These sensitivities are for Planck 143 GHz. The values should be also multiplied by $f_{\text{sky}}$.

$\text{f}_{\text{NL}}$ sensitivity with $T,E$
Secondary effects

- The ISW-lensing effect generates a non-Gaussian effect due to the same gravitational potentials that produce the ISW effect at large angular scales and the lensing one at the small ones.
- The ISW-lensing bispectrum is given by:

\[ b_{\ell_1\ell_2\ell_3}^{ISW-lensing} = \left\{ \frac{\ell_2(\ell_2 + 1) - \ell_1(\ell_1 + 1) - \ell_3(\ell_3 + 1)}{2} \right\} C_{\ell_1}^{TT} C_{\ell_3}^{TT} + 5 \text{perm} \]

![lensed power spectrum](image1)

![\(C_1^{\ psi}\) power spectrum](image2)
ISW-lensing and primordial bispectra

Figure 6. Contour plot of $l_1 l_2 l_3 (l_1 + l_2 + l_3) b_{l_1 l_2 l_3}$ (with non-linear intervals) for ISW-lensing (left) and local-model primordial non-Gaussianity (right; different overall scale). Although both are peaked for squeezed configurations, there are large phase and shape differences.

(from Lewis, Challinor & Hanson 2011)
Expected levels of detectability

Very optimistic. Not considered the NG signal due to the ISW-lensing

Realistic. Considered the NG signal due to the ISW-lensing

See Lewis, Challinor & Hanson (2011) for more details
Expected levels of detectability

The sky cut also reduces this value by a factor of $\sim f_{\text{sky}}$. 

Very optimistic. Not considered the NG signal due to the ISW-lensing

Realistic. Considered the NG signal due to the ISW-lensing

See Lewis, Challinor & Hanson (2011) for more details
Expected bias on local $f_{nl}$

$$\Delta f_{nl} = \sum_{\ell_1 \leq \ell_2 \leq \ell_3 \leq \ell_{\text{max}}} \frac{B_{\ell_1 \ell_2 \ell_3}^{\text{ISW-lens}} B_{\ell_1 \ell_2 \ell_3}^{\text{prim}}}{C_{\ell_1} C_{\ell_2} C_{\ell_3}} \sum_{\ell_1 \leq \ell_2 \leq \ell_3 \leq \ell_{\text{max}}} \frac{B_{\ell_1 \ell_2 \ell_3}^{\text{prim}} B_{\ell_1 \ell_2 \ell_3}^{\text{prim}}}{C_{\ell_1} C_{\ell_2} C_{\ell_3}}$$

<table>
<thead>
<tr>
<th></th>
<th>$\sigma_{f_{NL}}$</th>
<th>$\sigma_{\text{lens}}$</th>
<th>correlation</th>
<th>bias on $f_{NL}$</th>
<th>$\sigma_{f_{NL}}^{\text{marg}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>4.31</td>
<td>0.19</td>
<td>0.24</td>
<td>9.5</td>
<td>4.44</td>
</tr>
<tr>
<td>T+E</td>
<td>2.14</td>
<td>0.12</td>
<td>0.022</td>
<td>2.6</td>
<td>2.14</td>
</tr>
<tr>
<td>Planck T</td>
<td>5.92</td>
<td>0.26</td>
<td>0.22</td>
<td>6.4</td>
<td>6.06</td>
</tr>
<tr>
<td>Planck T+E</td>
<td>5.19</td>
<td>0.22</td>
<td>0.13</td>
<td>4.3</td>
<td>5.23</td>
</tr>
</tbody>
</table>

Table 1. Errors and biases on CMB lensing and primordial local-model non-Gaussianity parameterized by $f_{NL}$ for Planck-like noise (assuming isotropic coverage over the full sky with sensitivity $\Delta T = \Delta Q/2 = \Delta U/2 = 50 \mu \text{K arcmin}$ [$N_t^T = N_t^Q/U/4 = 2 \times 10^{-4} \mu \text{K}^2$] and a beam FWHM of 7 arcmin) or cosmic-variance limited data with $\ell_{\text{max}} = 2000$. From eq. (5.35) the errors $\sigma_{f_{NL}}$ and $\sigma_{\text{lens}}$ are the errors on the amplitudes of the corresponding bispectrum templates individually when the other one is fixed; $\sigma_{f_{NL}}^{\text{marg}}$ is the Fisher error on $f_{NL}$ if the amplitude of the lensing contribution is marginalized over; and the correlation is that between the two bispectrum shapes. The bias is the systematic error on $f_{NL}$ if the CMB lensing contribution is neglected, i.e. eq. (5.26).
The ISW-lensing estimator

- The same estimators used for the primordial non-Gaussian bispectrum can be also used for the ISW-lensing one. However, the ISW-lensing signal is now significantly contributing to the bispectrum covariance matrix.
- Another approach is to first reconstruct the lensing potential $\psi$ from the CMB temperature map and then cross-correlated with the CMB anisotropies (Lewis et al. 2011):

$$
\hat{S} = \frac{1}{F} \sum_{\ell_1 m_1} C^{T\psi}_{\ell_1} \frac{\tilde{T}_{\ell_1 m_1}}{C_{TT}^{tot, \ell_1}} \hat{\psi}^*_{\ell_1 m_1} \frac{N^{(0)}_{\ell_1}}{N^{(0)}_{\ell_1}} = \frac{1}{F} \sum_{\ell_1} (2\ell_1 + 1) \frac{C^{T\psi}_{\ell_1} C^{T\psi}_{\ell_1}}{N^{(0)}_{\ell_1}}
$$

$$
F \approx \sum_{\ell_1} (2\ell_1 + 1) \left(C^{T\psi}_{\ell_1}\right)^2 \left(C_{TT}^{tot, \ell_1}\right)^{-1} \left(N^{(0)}_{\ell_1}\right)^{-1}
$$

$$
\left[N^{(0)}_{\ell_1}\right]^{-1} \equiv \frac{1}{2\ell_1 + 1} \sum_{\ell_2 \ell_3}^{\ell_1 \leq \ell_2 \leq \ell_3} \Delta^{-1}_{\ell_1 \ell_2 \ell_3} \left[ \frac{\tilde{C}_{TT}^{tot, \ell_3} F^0_{\ell_2 \ell_1 \ell_3} + \tilde{C}_{TT}^{tot, \ell_2} F^0_{\ell_3 \ell_1 \ell_2}}{\tilde{C}_{TT}^{tot, \ell_2} \tilde{C}_{TT}^{tot, \ell_3}} \right]^2
$$
Cubic statistics for ISW-lensing bispectrum

- Select a characteristic set of angular scales $R_j$
- Compute the wavelet transform map $w(b_i, R_j)$ at these scales $R_j$
- Compute third order statistics by combining different scales
  \[ q_{ijk} = \frac{1}{\sigma_i \sigma_j \sigma_k} \int w(\vec{b}, R_i) w(\vec{b}, R_j) w(\vec{b}, R_k) d\vec{b} \]
- Combine in a $\chi^2$ test all the estimators
  \[ \chi^2(A) = \sum_{ijk, rst} (q_{ijk} - \langle q_{ijk} \rangle_A) C^{-1}_{ijk, rst}(A) (q_{rst} - \langle q_{rst} \rangle_A) \]

Because there is guaranteed non-Gaussian signal present due to the ISW-lensing, the covariance is larger and it should be computed as a function of A (See Lewis et al 2011)
The extragalactic contamination

- The extragalactic background of unresolved sources (radio plus IR galaxies) produces a bias in the estimate of $f_{NL}$ and a slight increase in its error bar.
- Number counts are based on up-to-date models of populations of radio galaxies (Tucci et al. 2011) and far-IR galaxies (Lapin et al. 2011).
- The $f_{NL}$ bias and error bar are given by the expressions:

$$\Delta f_{nl} = \sigma^2 \sum_{\ell_1 \leq \ell_2 \leq \ell_3 \leq \ell_{\text{max}}} \frac{1}{\Delta \ell_1 \ell_2 \ell_3} \frac{B_{\ell_1 \ell_2 \ell_3} B'_{\ell_1 \ell_2 \ell_3}}{C_{\ell_1} C_{\ell_2} C_{\ell_3}}$$

$$\frac{1}{\sigma^2} = \sum_{\ell_1 \leq \ell_2 \leq \ell_3 \leq \ell_{\text{max}}} \frac{1}{\Delta \ell_1 \ell_2 \ell_3} \frac{B'_{\ell_1 \ell_2 \ell_3} B'_{\ell_1 \ell_2 \ell_3}}{C_{\ell_1} C_{\ell_2} C_{\ell_3}}$$

where $C_\ell = \left(C_\ell^{\text{CMB}} + C_\ell^{\text{PS}} \right) b_\ell^2 + C_\ell^{\text{noise}}$

- The bias obtained are relatively small for Planck:
  - $T$: $\Delta f_{NL}^{\text{loc}} = 0.3, \Delta f_{NL}^{\text{equi}} = 15, \Delta f_{NL}^{\text{ort}} = 1.7$ (see also Lacasa et al. 2011 for $\Delta f_{NL}^{\text{loc}}$)
  - $T+E$: $\Delta f_{NL}^{\text{loc}} = 0.03, \Delta f_{NL}^{\text{equi}} = 5.4, \Delta f_{NL}^{\text{ort}} = 2.0$
Summary

- Non-Gaussianity studies represent an important complement to the power spectrum to constrain inflationary models.
- Optimal estimators of $f_{NL}$ are implemented in different spaces (harmonic, wavelets, modal). They are an important test to check for systematics and foreground contamination.
- The ISW-lensing bispectrum is expected to be detected at the 3-4σ with Planck. It also produces a bias in the local primordial shape of $\Delta f_{NL}$ =7-9
- The extragalactic point sources bias the value of $f_{NL}$ by a relatively small amount (smaller than the expected error bar for each shape). Including polarization the bias is decreased.
- When estimating $f_{NL}$ all possible contributions should be considered (primordial, ISW-lensing, extragalactic point sources, Galactic foregrounds).
- Planck is expected to improve WMAP results on $f_{NL}$ by a factor of approx. 5.