Inpainting and Morphological Component Analysis

Sandrine Pires
sandrine.pires@cea.fr

AIM-CEA Saclay, France
Methods based on sparsity

Considering a transform: \( \alpha = \Phi^T X \)

A signal \( X \) is sparse in a basis \( \Phi \) if most of the coefficients \( \alpha \) are equal to zero or close to zero.
Signal and image representations

- Local DCT:
  - Stationary textures
  - Locally oscillatory

- Wavelet Transform
  - Piecewise smooth
  - Isotropic structures

- Curvelet Transform
  - Piecewise smooth
  - Edge structures
Morphological Component Analysis

(Starck et al, 2004)

\[ Y = X_1 + X_2 \]

\[ \alpha_i = \phi_i^T X_i \]
Morphological Component Analysis

\[
\min_{\alpha_i} \sum_i \| \alpha_i \|_1 \quad \text{s.t.} \quad Y = \sum_i \Phi_i \alpha_i
\]

Ridgelet component

DCT component

Contour image

Texture
Morphological Component Analysis
Morphological Component Analysis

Wavelet component
Compact sources

Curvelet component
Filaments
Morphological Component Analysis
Morphological Component Analysis

Wavelet component

Ridgelet + Curvelet component
Morphological Component Analysis

Original image

Reconstructed image
MCA TUTORIAL
Missing data
(Elad et al, 2005)

✓ Causes of missing data:
   ✓ Occurrence of defective or dead pixels
   ✓ Partial sky coverage due to problems in the scan strategy
   ✓ Saturated pixels
   ✓ Absorption or masking of the signal by a foreground

✓ Problems caused by missing data:
   ✓ Bias and decrease on statistical power
   ✓ Distortions in the frequency domain due to abrupt truncation
   ✓ Other edge effects in multi-scale transforms

✓ How to deal with missing data?
   ✓ Correction of the measure by the proportion of missing data
   ✓ Other corrections specific to a given measure (i.e. MASTER for power spectrum estimation)
   ✓ Inpainting methods
Inpainting based on sparsity

\[
\min_{\alpha} \|\alpha\|_1 \quad \text{s.t.} \quad Y = M \Phi \alpha
\]
Inpainting on asterosismic data

Light curve (time series)

Zoom on the Light curve

Power spectrum

Original (red) and masked (black) data

Inpainted data (black)
Missing data
In Weak Lensing
Missing data
In Weak Lensing data
Inpainting in Cosmic Microwave Background data
Inpainting in Cosmic Microwave Background data
INPAINTING TUTORIAL