Constraints on local primordial non-Gaussianity using a neural network classifier

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Outline

Constraints on local non-Gaussianity using a neural network classifier
Casaponsa B., Bridges M., Curto A., Barreiro R.B., Hobson M.P., Martínez-González E.

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Local non-Gaussianity

Gaussianity

Standard inflationary model $\mapsto$ Gaussian distribution of the anisotropies

Non-Gaussianity

Any deviation from normal probability distribution. Different processes can show different deviations.

local fnl parameter

$\phi = \phi_L + f_{NL}\left[\phi_L^2 - <\phi_L^2>\right] \Rightarrow \frac{\Delta T}{T} = F(\phi, f_{NL})$.

Third order moments, as for example the bispectrum, are linearly dependant to fnl. $f_{NL} \propto <(\frac{\Delta T}{T})^3>$

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Very weak signal!

Method's efficiency:

- in terms of accuracy = bispectrum (Smith et al. 2009, Komatsu et al.2011, SMHW Curto et al. 2011a,b)
- CPU time (SMHW Curto et al. 2011a,b)

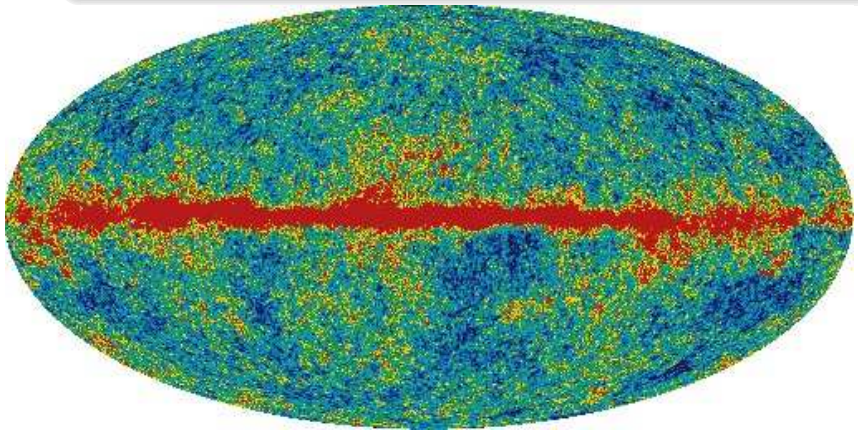
Aim of this work

- We want to show that using neural networks we are able to get equivalent results as the ones obtained through χ^2 minimization, avoiding C estimation and invertion

Estimators

REAL DATA

f_{NL} estimators get more complicated when including the mask and anisoptropic noise.



http://map.gsfc.nasa.gov/

The optimal estimator has been proposed by Creminelli et al. (2006) and succesfully computed by Smith et al. (2009) and Komatsu et al.(2011) for WMAP-5year and WMAP-7year data, giving the best constraints until the moment. $-10 < f_{NL} < 74$.

$$\hat{f}_{NL} = \frac{1}{N} \sum_{l_i m_i} \Big(\langle a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3} \rangle_1 \; C^{-1}_{l_1 m_1, l_4 m_4} C^{-1}_{l_2 m_2, l_5 m_5} C^{-1}_{l_3 m_3, l_6 m_6} a_{l_4 m_4} a_{l_5 m_5} a_{l_6 m_6}$$

$$-3 \, \langle a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3} \rangle_1 \; C^{-1}_{l_1 m_1, l_2 m_2} C^{-1}_{l_3 m_3, l_4 m_4} a_{l_4 m_4} \Big) \, ,$$

where N is a normalization factor:

$$N = \sum_{l_i m_i} \langle a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3} \rangle_1 \; C^{-1}_{l_1 m_1, l_4 m_4} C^{-1}_{l_2 m_2, l_5 m_5} C^{-1}_{l_3 m_3, l_6 m_6} \langle a_{l_4 m_4} a_{l_5 m_5} a_{l_6 m_6} \rangle_1 \, . \quad (2)$$

SMHW analysis using simulations have also found similar results (Curto et al. 2011) and there is no need of a linear term $-16 < f_{NL} < 76$.

$$\hat{f}_{NL} = \frac{\sum S_{ijk}^{fnl=1} C_{ijk,rst}^{-1} S_{rst}^{obs}}{\sum S_{ijk}^{fnl=1} C_{ijk,rst}^{-1} S_{rst}^{fnl=1}}$$

Where $S \sim w_{l_1 m_1} w_{l_2 m_3} w_{l_3 m_3}$ are third order moments combining different wavelet scales.

These methods are computationally demanding and the covariance matrix and its inverse has to be estimated (large number of simulations, singular matrix).

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We want to bypass this last step using neural networks.

Neural networks

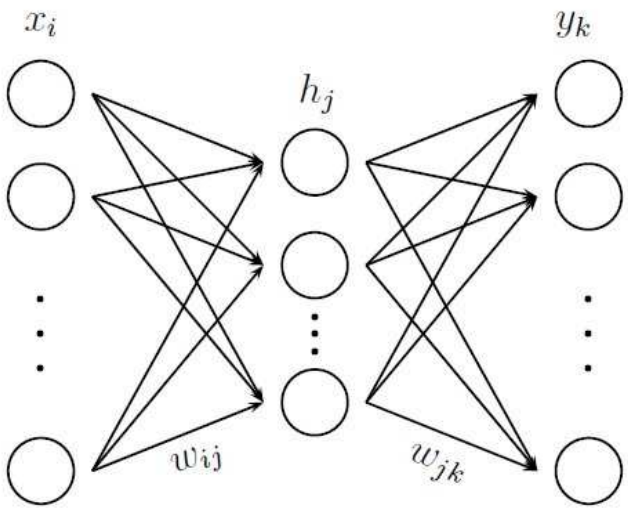

Figure 1. Schematic of a 3-layer feed-forward neural network.

nodes

$$y_k = \sum_j w_{kj} h_j + \theta_k ,$$

where h_j is

$$h_j = tanh(\sum_i w_{ji} x_i) + \theta_j$$

$$y_k = \sum_j w_{kj} \left(\tanh(\sum_i w_{ji} x_i) + \theta_j \right) + \theta_k$$

Supervised training for a feedforward network

We train the network with a known set of inputs and outputs, $\mathbf{x^t}$ and $\mathbf{y^t}$. We choose an optimization function (Ex. mse,rmse,χ^2,...) .

The optimization function is only dependent of the network parameters.

$$\chi = \frac{1}{2} \sum_{t,k} (y_k^{(net),t} - y_k^{(t)})^2$$

minimize this function (using conjugates gradient methods, gradient descent method, etc.)

We have used a neural network code with $Q = \alpha S - \chi^2$, where S is the entropy. Following the maximum entropy trajectory to find the optimal solution (MEMSYS, Gull and Skilling 1999). In any case we need to find w_{lm} and $\theta_n \mapsto y_k \sim y_k^{real}$.(Mike Hobson's talk)

Neural Network classifier

$$p_k = \frac{e^{y_k}}{\sum_k e^{y_k}}$$

$$\chi = \sum_k p_k \ln p_k^{(net)}$$

Training

- We need to know how many classes we want and what they represent
- Supervised training requires known $\vec{x}^t$ and $\vec{p}^t$.
- Inputs need to be the characteristic properties of the objects we want to differentiate

Ex. we want to classify apples and oranges. 

1. we need to have a sample of these two fruits, N apples and oranges.
2. we choose the best properties to differenciate them, ex: color, acidity, texture...
3. we introduce this values to the network.
   - INPUTS $\mapsto$ fruit's properties
   - OUTPUTS $\mapsto$ probability each class (ex. p=(1,0))

After the training using N fruits we will get the weights and biases that should be able to generalize the problem. We can put the properties of any orange / apple and the network ouputs will be the probablility of belonging to each class.

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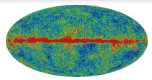
What we really want to classify are CMB maps in levels of non-Gaussianity. 

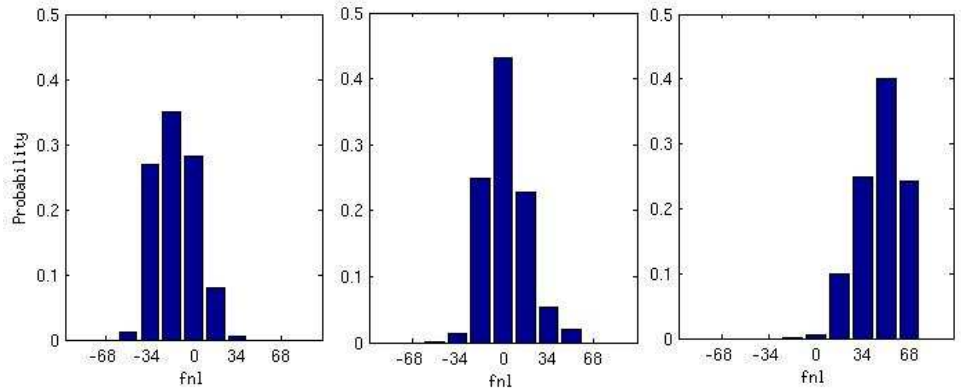
1. Sample: CMB simulated maps with different values of f_{NL}.

   $$a_{lm} = a_{lm}^{(G)} + f_{NL} a_{lm}^{(NG)}$$

   (Elsner & Wandelt, http://planck.mpa-garching.mpg.de/cmb/fnl-simulations)
2. Characteristic properties: Third order moments (cubic statistics of the wavelet coefficients)
   $S_{jkl} = \sum_i \frac{w_j w_k w_l}{N_{pix}} \mapsto$ 680 inputs
3. Outputs: Different levels of non-Gaussianity. (ex. $-100 < f_{NL} <= -80$ class 0, $-80 < f_{NL} <= -60$ class 1,...) $\mapsto$ 9 classes

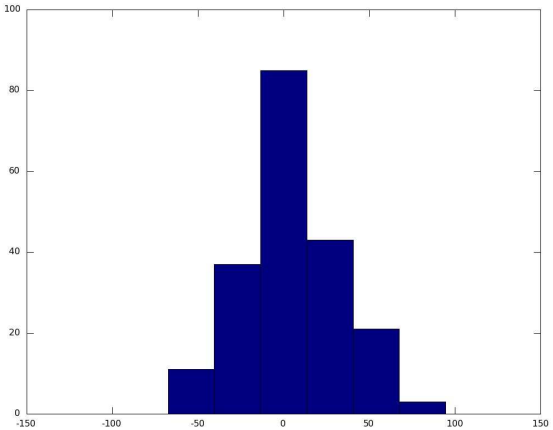
After training and testing the network gives the probability of an input vector (statistics of the CMB map) to belong at each class p_i.

Neural Network classifier



We can compute f_{NL} for a given map as:

$$\hat{f}_{NL} = \sum_i fnl_i^{(c)} \times p_i$$



If we do that for 1000 simulations with same fnl, we can compute the bias and dispersion of the estimator $\mapsto$ the efficiency of the method.

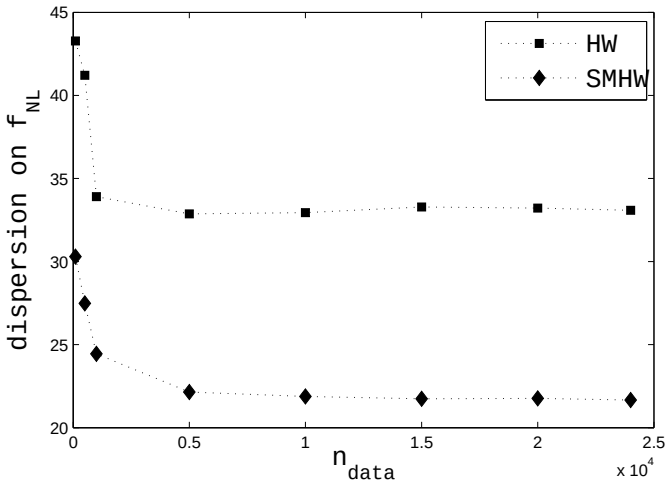
Problems

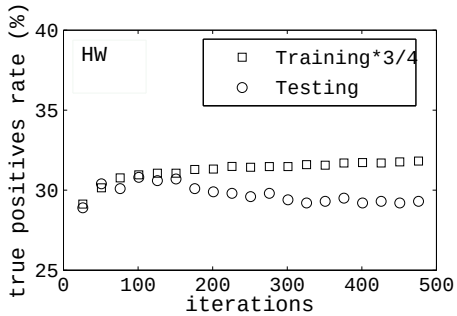
- Working with repeated a_{lm} realizations makes overfitting very likely
- We detect this when the testing and the training set have divergent behaviour.

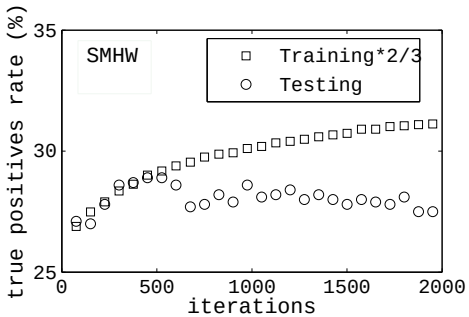
Results

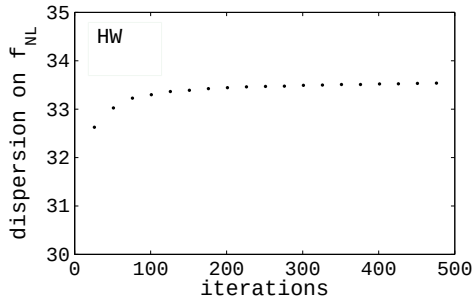
Stopping at the moment where the overfitting starts

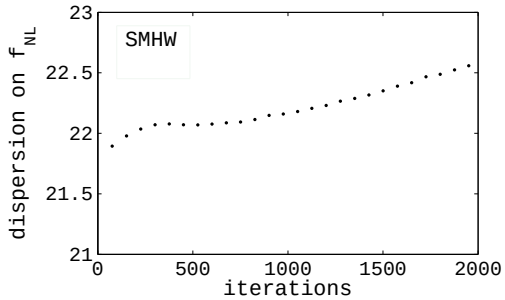
$$\text{True positives rate} = \frac{\text{Right classified inputs}}{\text{Total of inputs}}$$











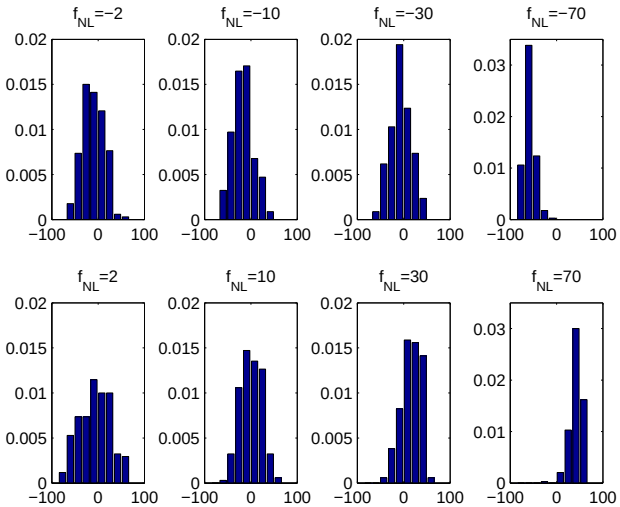
Checking how affects number of inputs to train

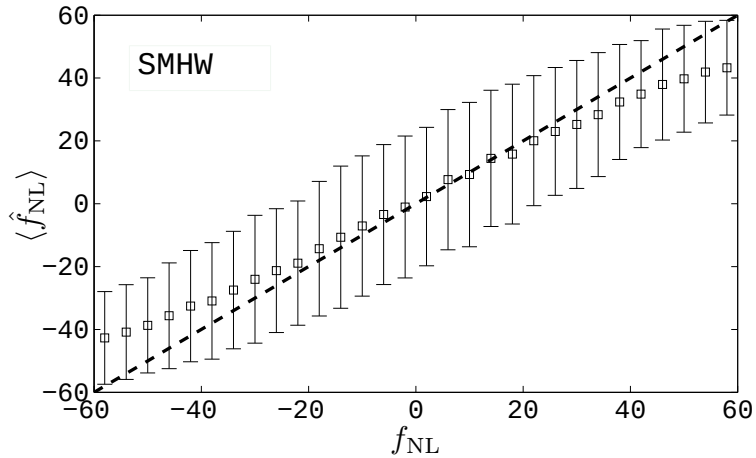
Dispersion computed for $N_{test} = 1000$. $N_{train} = 5000$.

Results

Distribution of $\hat{f}_{NL}$

No significant bias. Edge problem for large f_{NL} (NN can not give values outside training range)





Results

	$\hat{f}_{NLdata}$	$\sigma(\hat{f}_{NL})$	$< \hat{f}_{NLgauss} >$	$P_{2,5}$	$P_{97,5}$
SMHW (NN)	19	22	-1	-43	42
SMHW (WLS) Curto et al. 2011b	32	21	0	-42	46
HW (NN)	-12	33	-1	-66	63
HW (WLS) Casaponsa et al. 2011	6	34	1	-68	67

Results obtained with neural networks (NN) and weighted least squares (WLS). $\hat{f}_{\rm NLdata}$ is the best fitting value for V+W WMAP data, $< \hat{f}_{\rm NLgauss} >$ and $\sigma(\hat{f}_{\rm NL})$ are the expected value and the standard deviation for Gaussian simulations. $P_{2,5}$ and $P_{97,5}$ represent the percentile values at 95 % confidence level of $\hat{f}_{\rm NL}$ for Gaussian realizations.

Conclusions

- Neural network estimator of f_{NL} using wavelets coefficients gives same results avoiding the inversion of the covariance matrix.
- Neural networks might be useful in other cases where matrix invertions are involved.
- We have to be careful with overfitting and network architecture.
- Once the network is trained (in this specific case no more than 1 minute) generalized results are immediate. Point sources, assymetries, etc. can be calculated if simulations available.
- Next step bispectrum.

Thanks