MAP-based sparse detection strategies. 
Application to the hyperspectral data of the MUSE instrument.

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ADA 7, Cargèse  
15 May 2012
We consider two detection tests adapted to sparse vector parameters:

- Simple introductory model;
- More realistic model using a redundant dictionary.

Application to detection of specific features in MUSE hyperspectral data.

Contributions:

- The proposed tests are more efficient than the classical ones (GLR).
- Detection strategy exploiting the spatial dependencies between the spectra in MUSE hyperspectral data.
- Analysis of the resulting global False Alarm (FA) rate, through the use of FA-maps.
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Considered Tests

- **General Detection Test**: $T(x: data) \overset{H_1}{\geq} \gamma_{H_0}$

- **Considered Tests**:
  
  Posterior Density Ratio $^1$:
  
  $$PDR(x) = \frac{\max p(\theta|x)}{p(0|x)}$$

  Likelihood Ratio using $\hat{\theta}|H_1 = \hat{\theta}_{MAP}$:
  
  $$LR_{MAP}(x) = \frac{p(x|\hat{\theta}_{MAP})}{p(x|0)}$$

  Generalized Likelihood Ratio:
  
  $$GLR(x) = \frac{\max p(x|\theta)}{p(x|0)}$$

  PDR and LR_{MAP} tests should favor sparsity thanks to the MAP.

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  - **Likelihood Ratio using \( \hat{\theta}|\mathcal{H}_1 = \hat{\theta}_{\text{MAP}} \)**:
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Simple introductory model

Hypothesis Test:
\[
\begin{cases}
\mathcal{H}_0 : x = \epsilon, & \epsilon \sim \mathcal{N}(0, \Sigma) \\
\mathcal{H}_1 : x = \theta + \epsilon
\end{cases}
\]

Test Statistics:

\[
T_{LRMAP}(x) = \sum_{i=1}^{N} \left( \frac{x_i^2}{\sigma_i^2} - \eta^2 \right) I\left( |x_i|/\sigma_i > \eta \right)
\]

\[
T_{PDR}(x) = \sum_{i=1}^{N} \left( |x_i|/\sigma_i - \eta \right)^2 I\left( |x_i|/\sigma_i > \eta \right)
\]

- \( \theta \) unknown, deterministic and sparse;
- \( \Sigma = \text{diag}(\sigma_1^2, \cdots, \sigma_N^2) \).
- Laplacian prior: \( \pi(\theta) = \prod_i \frac{1}{2\lambda_i} e^{-|\theta_i|/\lambda_i} \);
- \( I(\cdot) \) : Indicator Function;
- Equal per component threshold \( \eta \ \forall x_i \);
- \( T(x, \eta) > \gamma : P_{FA} \) and \( P_{DET} \) depend on both the \( \eta \) and \( \gamma \) thresholds.
- \( T_{LRMAP}|_{\mathcal{H}_0} : \) sum of \( N \) truncated \( \chi^2_1 \) translated by \( \eta \) with probability mass in 0.
- Expression of \( P_{FA} \) under 'rough' Gaussian approximation.
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Expression of \( P_{FA} \) under 'rough' Gaussian approximation.
Comparative Analysis : PDR vs LRMAP

- Performances of the tests varying the $\eta$ and $\gamma$ thresholds:

  \begin{itemize}
  \item $\eta = 0.05$
  \begin{itemize}
    \item PDR $\simeq$ LRMAP
  \end{itemize}
  \item $\eta = 1$
  \begin{itemize}
    \item PDR $> \text{LRMAP}$
  \end{itemize}
  \item $\eta = 3.3$
  \begin{itemize}
    \item PDR $< \text{LRMAP}$
  \end{itemize}
  \end{itemize}

- PDR (red) and LRMAP (blue) ROC curves compared to the GLR’s one (green).
  - PDR and LRMAP tests outperform the GLR;
  - PDR and LRMAP performances can be inverted.
To overcome the problem of the PDR and LRMAP tests parameter dependency, we set $\gamma = 0$.

In this case:

- the tests depend only on $\eta$ that fixes the $P_{FA} \triangleq P_{FA_0}$:

$$P_{FA_0} = Pr(T(x) > 0|\mathcal{H}_0) = 1 - (2\Phi(\eta) - 1)^N,$$

with $\Phi(\cdot)$: cumulative distribution function of a standard normal distribution;

- the two tests $T(x) > 0$ are the same since both test statistics are non zero if at least one component is above the threshold $\eta$.

- We refer to this unique test as the PDR/LRMAP test.
Setting $\gamma = 0$

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Hypothesis Test:

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\begin{align*}
\mathcal{H}_0 : x &= w, \quad w \sim \mathcal{N}(0, I) \\
\mathcal{H}_1 : x &= D\theta + w
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- **D**: Normalized highly redundant dictionary ($N \times L$ with $L \gg N$);

Test Statistics

- $T_{GLR} = \|x\|^2$;
- $T_{PDR}(x, \eta) = \frac{1}{2} \hat{x}_{MAP}^t \hat{x}_{MAP}$;
- $T_{LRMAP}(x, \eta) = \eta \|\hat{\theta}_{MAP}\|_1 + \frac{1}{2} \hat{x}_{MAP}^t \hat{x}_{MAP}$.

False Alarm rate for PDR/LRMAP test

$P_{FA_0} = Pr(T > 0 | \mathcal{H}_0) = Pr(\max_i(||D_i^t x|| > \eta | \mathcal{H}_0)$

- The GLR reduces to an *Energy Detector*;
- $\hat{x}_{MAP} = D\hat{\theta}_{MAP}$;

PDR and LRMAP are $\eta$ and $\gamma$ dependent.

- Simplification: $\gamma = 0 \Rightarrow P_{FA} = P_{FA_0}(\eta)$.

→ Computationally simple (and efficient) for massive data
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More realistic model: Sparsity in a redundant dictionary

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Multi Unit Spectroscopic Explorer

Based on the **Integral Field Spectroscopy** concept.

First light at the **VLT** in 2012

Huge quantity of Hyperspectral data

(300 × 300 × 3464)
Adapted Redundant Dictionary

- Designed for the astrophysical characteristics of the spectra to be observed by MUSE $^4$.
- Composed by three sub-dictionaries:

  - $R^\ell$ = Spectral Lines $\rightarrow$ Discrete splines of different widths;
  - $R^b$ = Spectral Breaks $\rightarrow$ Heaviside step functions;
  - $R^c$ = Continuous Spectra $\rightarrow$ Low frequencies sine functions.

$L = 22560$ atoms, $N = 3463$ spectral channels.

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MUSE Data Model

MUSE data model:
\[ H_1 : y = HR\alpha + \epsilon, \quad \epsilon \sim N(0, \Sigma) \]

- Considering weighted data:
  - \( x = \Sigma^{-\frac{1}{2}}y \);  
  - \( D_{\Sigma} = \Sigma^{-\frac{1}{2}}HR \rightarrow D = D_{\Sigma}N_{D_{\Sigma}}^{-1} \); 
  - \( \theta = N_{D_{\Sigma}}\alpha \); 

Equivalent model:
\[ H_1 : x = D\theta + w, \quad w \sim N(0, I) \]

- \( H \): Matrix form of the LSF (N×N); 
- \( R \): Redundant dictionary (N×L, L>N); 
- \( \alpha \): Sparse synthesis coefficients vector.

Tests implementation at \( \gamma = 0 \):
\( P_{FA} \) fixed by \( \eta \).

Correspondence \( \eta \leftrightarrow P_{FA} \) evaluated numerically.
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MUSE Cube \((14 \times 10 \times 2048)\);

- \(P_{FA_0} = 0.01\) for both tests;
- GLR : 10 detections;
- PDR/LRMAP : 22 detections.

For same \(P_{FA}\), increased \(P_{DET}\) for PDR/LRMAP on MUSE data thanks to the adapted redundant dictionary and the injection of the MAP in the tests.
Detection comparison on a simulated MUSE sub-cube

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Exploiting Spatial Dependencies between the spectra

Idea: *Cascade of two detection tests*

- After a *first detection round* using PDR/LRMAP test we are left with:
  - Detected
  - Not Detected

- We define the *contour*: set of non-detected pixels but contiguous to detected ones

- We run a *second detection round* on each pixel of the contour, taking into account spatial dependencies between contiguous spectra of the hyperspectral sub-cube.

We refer to this second test as the **LR-MPβ** test.

**Fig.**: MUSE sub-cube (14×10×2048).
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  ![Detected Not Detected](image)

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**Fig.**: MUSE sub-cube (14×10×2048).

**Fig.** shows the MUSE sub-cube with a detected region in green and a contour in brown.
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Fig. : MUSE sub-cube (14×10×2048).
LR-MP $\beta$ Test

- We define \textbf{Faint Contour Spectrum} $x_f$ (blue cross)
  \begin{itemize}
  \item → \textbf{Bright Detected Spectrum} $x_b$ (green cross)
  \end{itemize}

  where $x_f$ and $x_b$ are contiguous spectra in the cube.

- Under $H_1$, $x_f$ is modeled as: $x_f = \hat{x}_f + w$ where

  \[
  \hat{x}_f = \hat{\beta}_{ML} D \hat{\theta}(x_b),
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  with

  - $\hat{\beta}_{ML}$: amplitude coefficient obtained by ML estimate;
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  \text{LR} - \text{MP}\beta = \frac{p(x_f; \hat{\beta}, \hat{\theta}_b)}{p(x_f; 0)} \frac{H_1}{H_0} \xi
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  $$LR - MP\beta = \frac{p(x_f; \hat{\beta}, \hat{\theta}_b)}{p(x_f; 0)} \begin{cases} \frac{H_1}{H_0} \xi \\ \frac{H_0}{H_1} \xi \end{cases}$$
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  (green cross)

where $x_f$ and $x_b$ are contiguous spectra in the cube.

- Under $H_1$, $x_f$ is modeled as: $x_f = \hat{x}_f + w$ where

$$\hat{x}_f = \hat{\beta}_{ML} D\hat{\theta}(x_b),$$

with

- $\hat{\beta}_{ML}$ : amplitude coefficient obtained by ML estimate;
- $\hat{\theta}(x_b)$ : bright spectrum parameters vector computed by Matching Pursuit.

$$LR - MP\beta = \frac{p(x_f; \hat{\beta}, \hat{\theta}_b)}{p(x_f; 0)} \overset{H_1}{\gtrless} \xi$$
**Results on a MUSE sub-cube**

- **Improved detection performances for the PDR/LRMAP + LR-MP$\beta$ test.**

  - **PDR/LRMAP**: 22 detections;
  - **PDR/LRMAP + LR-MP$\beta$**: 34 detections.

- **The additional 12 detected spectra are true detections.**
$P_{FA}$ maps

- $P_{FA}$ of the cascade of the two tests:
  for one spectrum $x_f$ (contiguous to one bright spectrum $x_b$),

$$P_{FA}^{2\text{tests}}(x_f) = P_{FA0}(\eta) + [1 - P_{FA0}(\eta)]P_{DET0}(x_b, \eta)P_{FA}(x_b, \xi).$$

- The $P_{FA}$ increases with respect to the case where only one test is performed but, in any case

$$P_{FA}^{2\text{tests}} \leq 2P_{FA0}.$$

- Simulation results:
  - $P_{FA}^{1\text{test}} = 0.01$
  - $0.008 < P_{FA}^{2\text{test}} < 0.023$
  - mean $P_{FA}^{2\text{test}} = 0.014$
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Conclusions

Two new MAP-based tests for sparse signal detection were considered:

- the PDR test;
- the LRMAP test.

These detection tests were applied to astrophysical hyperspectral data.

- The tests were set in order to take advantage of a redundant dictionary and instrumental specificities, while keeping the processing complexity low.

- We proposed a new detection strategy based on the exploitation of the spatial dependencies existing between the spectra of the considered hyperspectral data.

- We proposed an analysis of the resulting global FA-rate through the use of FA-maps.

Perspectives

- Aggregation of the detected spectra in “objects” through unsupervised spatial-spectral clustering algorithms.

- Injection of others “thresholding” priors than Laplacian.
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Thank You.
Results on a single MUSE spectrum

Spectrum considered:

- \( N = 2048 \rightarrow \# \) Wavelengths considered.
- \( \text{SNR} = 10 \log_{10} \frac{\|s\|^2}{\|\epsilon\|^2} = -19.3dB. \)

a) ROC curves of the tests.

b) Performances at variable SNR:
   - \( P_{FA_0} = 0.01 \) fixed (\( \eta = 4.72 \)).
   - GLR reduced to an ”Energy Detector”.
   - Better performances PDR/LRMAP vs GLR.
Expression of the $P_{FA}$ considering the 8 neighbors of a given spectrum:

$$P_{FA_{TOT}} = P_{FA_0} + (1 - P_{FA_0}) \times \frac{1}{8} \sum_{i=1}^{8} P_{DET_0}(x_{b_i}) P_{FA}(\hat{x}_{b_i}; \xi)$$