High quality CMB map estimation

J. Bobin, J. L. Starck and F. Sureau

CEA - Service d’Astrophysique, France
After a series of successful surveys such as COBE or WMAP:

- The CMB is fundamental to study the dawn of our universe!

- PLANCK provides full-sky data in 9 channels in the range 30GHz - 857GHz

- High resolution data of (up to 5 arcmin)

Will be the reference full-sky data in the next decades for CMB studies!

Raising various image processing problems...
CMB data analysis

The importance of Source Separation
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Extra foreground are superimposed with the CMB !!!
Point sources, galactic foregrounds, ... etc
Standard approaches amount to model each observation as a linear mixture of elementary components (i.e. CMB, SZ, Synchrotron, Free-Free, Dust ...) :

\[ \forall i; x_i = \sum_j a_{ij} s_j + n_j \]

Which can be recast as:

\[ X = AS + N \]
CMB data analysis

Separate the wheat and the chaff!

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The objective is to estimate both A and S simultaneously!!
This inverse problem is classically known as Blind Source Separation (BSS)

\[ \mathbf{X} = \mathbf{A}\mathbf{S} + \mathbf{N} \]

Due to its particular structure (e.g. bilinearity ...), BSS is an ill-posed problem!

Standard methods mainly differ in the way they try to differentiate between the sources S. State-of-the-art methods in astrophysics include:

- **SMICA**: second-order statistics in the sph. harmonics space (rely on the differences of the components’ power spectra)
- **CCA**: higher-order statistics to enforce independence (ICA)
- **GMCA**: sparsity of the components in wavelets

Other approaches include Internal Linear Combination (ILC), parametric methods template fitting.
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Finding the best parameters \((A,S)\) so that the resulting components are the sparsest should provide an efficient separation process!

More formally ...

\[
\{ A, S \} = \text{Argmin}_{A,S} \sum_{j} \lambda_j \| s_j W \|_1 + \| X - AS \|_{F,\Sigma}^2
\]
Several important components can be well approximated by a rank-1 contribution:

- CMB, SZ and Free-Free emission: their electromagnetic spectrum is assumed to be known \((i.e.\ the\ related\ columns\ of\ A\ are\ known\ and\ fixed)\)

- Synchrotron emission: rank-1 assumption / its electromagnetic spectrum is a power law with an unknown spectral index \((which\ can\ be\ estimated\ online\ within\ GMCA)\)
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- Dust emissions: simple model involve two cold/hot modified black-body emissions with spatially varying parameters

- Point sources have their own spectrum ...
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A single mixing matrix: not enough degrees of freedom! Mixtures vary spatially
Beyond GMCA

Let’s observe the observations ...

Observation
Beyond GMCA

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Observation

1 - The large of structures of some components (Synchrotron and Dust emissions to only name two) can be modeled accurately by global rank-1 contribution.
Beyond GMCA

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1 - The large of structures of some components (Synchrotron and Dust emissions to only name two) can be modeled accurately by global rank-1 contribution.

2 - Variations of the spectral behavior of these components is likely to vary at smaller scales.
Beyond GMCA

Local Multiscale mixture model

Local Multiscale Mixture Model

\[ X_k[p] = \sum_{j=1}^{n} a^j_k[p] s_{j,k}[p] + N_k[p] \]
Classically, we apply GMCA at each patch and each scale:

$$\min_{A_k[p], S_k[p]} \sum_j \lambda_j \| s_{j,k}[p] \|_1 + \| X_k[p] - A_k[p]S_k[p] \|_F^2, \Sigma_k[p]$$
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Which patch size?
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$$A_k^{(1)}$$

$$A_k^{(2)}$$
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$$\min_{A_k[p], S_k[p]} \sum_j \lambda_j \| s_{j,k}[p] \|_1 + \| X_k[p] - A_k[p] S_k[p] \|_F^2, \Sigma_k[p]$$

The idea is designing a multichannel quadtree decomposition.
Gathering up all the pieces ...
Gathering up all the pieces ...

At each location, we can choose the best local estimator !
Gathering up all the pieces ...

\[ A_k^{(1)} \ldots A_k^{(s)} \]
L-GMCA

Gathering up all the pieces ...

From the decorrelation of CMB, noise and foregrounds:

\[ \sigma_y[k, p]^2 = \sigma_x[k, p]^2 + \sigma_n[k, p]^2 + \sigma_f[k, p]^2 \]

- CMB
- Noise
- Residuals
Gathering up all the pieces ...

\[ X_j[p] \]

\[ \hat{A}_k[p] = \min_s \sigma_{y(s)}[k,p]^2 \]

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CMB  Noise  Residuals

Choosing the mixing matrix that provides the CMB with the lowest variance!
Planck component extraction via L-GMCA:

1) Decompose the data into J wavelet scales

2) At each scale k, apply GMCA to compute

\[ A_k^{(1)} [p] \rightarrow A_k^{(s)} [p] \]

3) At each scale and each patch, choose the “best” estimators

4) Reconstruct the CMB map via inverse wavelet transform
For the sake of evaluation, L-GMCA has been applied to simulated but realistic data (Leach et al. 2008)

Planck sky modeling: CMB, SZ, free-free, synchrotron and dust emission, spinning dust

Instrumental modeling: decorrelated but non-stationary gaussian noise, perfect isotropic gaussian beams
CMB map estimation

Input CMB map

-0.50  0.50
CMB map estimation

N-ILC

N-ILC
CMB map estimation

L-GMCA

L-GMCA
CMB map estimation

Residual at 5 arcmin

NILC

-0.060  0.060
CMB estimation

Power spectra
Foreground contamination

Cross-powspec residual/dust emission

ILC
NILC
GMCA
L−GMCA
Foreground contamination

cross-spectra

Cross-powspec residual/sz emission

ILC
NILC
GMCA
L-GMCA
Non-Gaussianity

Kurtosis per wavelet scale

- ILC
- NILC
- GMCA
- L-GMCA
CMB map estimation

Kurtosis - 1st wavelet scale

- ILC
- NILC
- GMCA
- L-GMCA

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Take-away messages

The local/multiscale mixture we studied along with sparsity yields clearly improvements:

- Lower foreground contamination
- Lower NG contamination

Still a lot to be explored:

- Accounting for astrophysical models (foregrounds)
- Towards a true full-sky estimation of the CMB map
- Extension to polarized data