A Global Strategy for Spectral Inversion from a Model Library

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PURPOSE

• Software requirement for the analysis of the Gaia/RVS stellar spectra
• Statistical fitting from a model grid
• Specific algorithms
  – Conception, coding & tests
• Comparison on simulated data

A.Bijaoui, et al. *Parameter estimation from a model grid. Application to the Gaia RVS spectra*

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➔ A strategy for fitting from model grids
The Global Strategy

• Pre-Processing → Normalized data
• Principal component decomposition
  – Number of components depends on the SNR
• The minimum distance in log(N) time
  – Balanced oblique decision tree
• Parameter refinement with a variant of the Gauss-Newton algorithm
• Bias removal with a Bayesian analysis
  – Take into account previous simulations
A Model Grid Example

• Gaia/RVS:
  – 971 pixels
  – 3 parameters
  – 1638 spectra

• Simulated observations:
  – Random interpolation on the grid
  – SNR: infinity, 100 et 10
The restored grid from 33 components
The noise and the components
PCA application: the results

- The reduction of the handled coefficients improves the efficiency
- The reduction of the CPU time is < to the reduction of the array length.
- The data compression allows the application to larger model grids
  - Component increase < Models increase
- Allows the application to larger spectra
  - Component increase < Pixel increase
- The biases increase ➔ Has to be reduced
Minimum search with a decision tree

- Minimum search needs $N$ distances / observation
- How to reduce to $\log(N)$?
  - Dichotomic partition
  - $kd$-tree with the median
  - Balanced $kd$-tree
- What coordinate at each node?
  - Too many pixels / $N$
  - Pixel combination
- Balanced Oblique Decision Tree (BODT)
The first level decomposition

Level 1

Level 2

Node 2

Node 3
Recognition of a noisy model

selected models → minimum distance
DEGAS + Gauss-Newton

- Minimum distance with DEGAS
  - Variations due to the sampling
- Gauss-Newton iterative corrections
  - Local linear approximation
  - Need of the Jacobian $J$ Model / parameters
    \[
    \delta \Theta = (J^T J)^{-1} J^T (O - S)
    \]
  - Threshold the too large corrections
- The best tested algorithm
Bias corrections with a Bayesian analysis

- The model grid is \( \{ S_n(\Theta_n), n \text{ in } (1,N) \text{ and } O \text{ is a noisy observation vector} \)
- The posterior PDF (DPDF) is \( p(\Theta|\hat{\Theta}) \)
- The dispersion PDF \( q(\hat{\Theta}|\Theta) \)
  - Determined by numerical experiments
  - Often used instead of the PPDF
  - \( \Theta_0 \) is got, not the true parameter \( \Theta_0 \)
- The quality is got from the DPDF \( p(\Theta|\hat{\Theta}_0) \)
  which is not identical to the PPDF \( q(\hat{\Theta}|\hat{\Theta}_0) \)
- The difference can be important in case of estimation biases
Bias corrections with a Bayesian analysis

- Application of the Bayes’ rule

\[ p(\Theta | \hat{\Theta}_0) = \frac{q(\hat{\Theta}_0 | \Theta)P(\Theta)}{Q(\hat{\Theta}_0)} \quad Q(\hat{\Theta}_0) = \int q(\hat{\Theta}_0 | \Theta)P(\Theta)d\Theta \]

- The DPDF is approximated with Parzen’s windows

\[ q(\Theta | \Theta) = \sum_{k=1,K} \sum_{n=1,N} K(\Theta - \Theta_n, a) W(\Theta - \hat{\Theta}_{nk}, S) \]

- For a uniform prior we get approximatively the posterior mathematical expectation (Nadaraya-Watson’s formula)

\[ \overline{\Theta} = \frac{\sum_{n=1,N} \sum_{k=1,K} \Theta_{nk} W(\Theta - \hat{\Theta}_{nk}, S)}{\sum_{n=1,N} \sum_{k=1,K} W(\Theta - \hat{\Theta}_{nk}, S)} \]
Results

• The biases are removed !!
• Errors at SNR = 100 do not increased
• For SNR=10 the errors are reduced
  – Has to be deepened
• Slow procedure
  – Need at a lot of simulation vectors
  – Acceleration using a kd-tree or a Balanced Oblique Decision Tree
• The use of a post-estimation Bayesian analysis seems to be essential
Conclusion

• The association of (PCA)+(Decision Tree) + (Gauss-Newton) + (Bayesian post-reduction) is very efficient for fast model fitting

• The SPEGAM software
  – System for Parameter Estimation from Grid of Astrophysical Models
  – F90 partly translated in Java for many applications
  – Should be easy to be applied for new applications
  – Associated documentation in progress including
    • A handbook on model fitting from a model grid
    • A programmer guide
    • A program reference manual
    • A user Guide